

# Energy Distribution in a Schwarzschild-like Spacetime

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*Received April 25, 2006; accepted September 21, 2006*  
*Published Online: February 21 2007*

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In this paper, utilizing Møller's energy-momentum complex, we explicitly evaluate the energy and momentum density associated with a metric describing a four-dimensional, Schwarzschild-like, spacetime derived from an effective gravity coupled with a U(1) gauge field in the context of a D3-brane dynamics in the classical regime, i.e., between the asymptotic and the Planck regime.

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**KEY WORDS:** energy-momentum complex; effective gravity; D-branes.

**PACS numbers:** 04.20.-q, 04.20.Cv, 04.40.Nr, 11.25.Uv.

## 1. INTRODUCTION

The subject of the localization of energy still lacks an acceptable answer and continues to be one of the most interesting and challenging problems in General Relativity. For the solution of the problem many researchers have computed the energy as well as the momentum and angular momentum associated with various space-times. However, the different attempts of constructing an energy-momentum density do not yield a generally accepted expression. After Einstein first introduced energy-momentum complexes (Einstein, 1915a,b), a plethora of different energy-momentum complexes were constructed, including those of Tolman (1934), Landau and Lifshitz (1987), Papapetrou (1948), Bergmann and Thompson (1953), Goldberg (1958), Weinberg (1972) and Møller (1958). With the exception of Møller's approach, which could be utilized to any coordinate system, the above energy-momentum complexes have a drawback, as they give meaningful results only if the calculations are restricted to quasi-Cartesian coordinates.

In 1973, Ch. Misner, K. Thorne and J.A. Wheeler sustained that "anybody who looks for a magic formula for local gravitational energy-momentum is looking for the right answer to the wrong question" (Misner *et al.*, 1973). This is the actual

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meaning of the nonuniqueness of the pseudotensor for the energy-momentum. However, they concluded that the energy is indeed localizable only for spherical systems. A few years later, Cooperstock and Sarracino (1978) demonstrated that if the energy is localizable in spherical systems, then it is also localizable in any space-time.

In the 1990's H. Bondi sustained that "In relativity a nonlocalizable form of energy is inadmissible, because any form of energy contributes to gravitation and so its location can in principle be found" (Bondi, 1990). The idea of the energy-momentum complex was severely criticized for a number of reasons, such as the nontensorial nature of the energy-momentum complex and, hence, its dubious physical interpretation (Chandrasekhar and Ferrari, 1991), and the fact that different energy distributions were obtained by different energy-momentum complexes for the same geometry (Bergqvist, 1992a,b). Attempts to deal with the issue of the localization of the gravitational energy-momentum include also the quasi-local approach (Brown and York, 1993; Hayward, 1994; Chen and Nester, 1999; Penrose, 1982; Komar, 1959; Tod, 1983).

The issue of the energy-momentum localization by use of the energy-momentum complexes was revived by K.S. Virbhadra's pioneering work (Virbhadra, 1990a,b,c, 1992; Rosen and Virbhadra, 1993; Virbhadra and Parikh, 1993, 1994a,b; Virbhadra, 1995). In 1996, Aguirregabiria *et al.* (1996) showed that four different energy-momentum complexes (ELLPW, standing for Einstein, Landau-Lifshitz, Papapetrou, Weinberg) yield the same energy distribution for any non-static, spherically symmetric metric of the Kerr-Schild class. Furthermore, their results complied with the quasi-local mass definition given earlier by Penrose and Tod (Brown and York, 1993; Hayward, 1994; Chen and Nester, 1999; Penrose, 1982; Komar, 1959; Tod, 1983). In 1999, Chang *et al.* (1999) proved that every energy-momentum complex is associated with a legitimate Hamiltonian boundary term, thus supporting the quasi-locality of energy-momentum complexes and, hence, their acceptance.

The large number of interesting results recently obtained by many researchers point out that the energy-momentum complexes are powerful tools for evaluating the energy and momentum in a given space-time (Chamorro and Virbhadra, 1995, 1997; Xulu, 1998a,b; Vagenas, 2003b; Radinschi, 1999, 2000b; Yang and Radinschi, 2003a; Bringley, 2002; Súkeník and Sima, 2001; Sharif and Fatima, 2005a; Sharif, 2004b,a; Gad, 2004c, 2005a, 2004a; Patashnick, 2005; Salti, 2005d,b; Halpern, 2006). Important works are done with the energy-momentum complexes in 2- and 3-dimensional space-times (Vagenas, 2003c,d, 2005; Grammenos, 2005). Also, we point out some interesting papers (Korunur *et al.*, 2005; Salti, M. and Havare, 2005b; Salti and Havare, 2005c,a; Salti, 2005a; Aydogdu and Salti, 2005a,b; Salti, 2005c; Aydogdu *et al.*, 2005; Salti, 2006) which demonstrate that the (ELLPW)- and Bergmann prescriptions yield the same results as their tele-parallel gravity versions for a given spacetime.

In this work we have chosen Møller's prescription, because it is not restricted to quasi-Cartesian coordinates, as pointed out earlier. Furthermore, there are many results (Yang *et al.*, 1997, 1999; Vagenas, 2003a, 2004; Gad, 2004d, 2005b, 2004b; Havare *et al.*, 2005; Radinschi, 2000c; Sharif and Fatima, 2005b; Radinschi, 2000a,d, 2001a,b; Yang and Radinschi, 2004; Yang and Radinschi, 2003b; Yang and Radinschi, 2002; Radinschi and Yang, 2005; Radinschi and Grammenos, 2006) that recommend this prescription for the localization of energy. Thus, we implement the Møller prescription and calculate the energy density for a metric describing a Schwarzschild-like geometry in the classical regime, in the context of a D3-brane dynamics study. The calculations are performed with Mathematica and Maple, the latter having attached the GrTensor platform. Throughout the paper we used geometrized units ( $G = 1$ ,  $c = 1$ ) and let Greek indices run from 0 to 3. The remainder of the paper is organized as follows. In Section 2 we present the Schwarzschild-like spacetime, while in Section 3 we give a description of Møller's prescription. In Section 4 we explicitly determine the energy and momentum distributions in the Schwarzschild-like spacetime using Møller's prescription. Finally, Section 5 is devoted to a summary of the obtained results and concluding remarks.

## 2. THE SCHWARZSCHILD-LIKE GEOMETRY

In a recent work, Kar and Majumdar (2005) considered the evolution of gravity on a D3-brane in a noncommutative string theory. In particular, the authors relied on the fact, that a D3-brane world-volume incorporating Einstein's gravity coupled to the nonlinear theory of Maxwell may provide a framework for the formulation of an effective theory of quantum gravity at the Planck scale. In going towards the Planck regime, they combined a theory of effective gravity with a U(1) gauge field thus obtaining, in the classical regime, a Schwarzschild-like and a Reissner-Nordström-like solution in (3 + 1) dimensions. In this work, we focus on the Schwarzschild-like solution.

Specifically, the authors considered a Euclidean world-volume spanned by  $(y_1, y_2, y_3, y_4)$  with a signature  $(+, +, +, +)$ , where the Minkowski signature may be obtained by analytic continuation  $y_4 \rightarrow it$ . Starting from the asymptotic regime with a flat D3-brane one can generalize the brane's description by including a slow variation in the induced metric  $g_{\mu\nu}$  in going into the classical regime. The dynamics of the brane is then governed by the coupling of the general-relativistic action with an appropriate Dirac-Born-Infeld action, whereby the authors considered a static gauge condition on spacetime. Thus, the complete action becomes

$$S = \frac{1}{16\pi} \int d^4y \sqrt{g} R + S_{\text{DBI}} \quad (1)$$

with  $R$  the scalar curvature, and  $S_{\text{DBI}}$  the Dirac-Born-Infeld action. After expanding, one obtains

$$S = \int d^4y \sqrt{g} \left( \frac{1}{16\pi} R - \frac{1}{4} g^{\mu\nu} g^{\lambda\rho} \mathcal{F}_{\mu\lambda} \mathcal{F}_{\nu\rho} + \mathcal{O}(\mathcal{F}^4) + \dots \right) \quad (2)$$

where  $\mathcal{F}_{\mu\nu}$  is the U(1) gauge field and higher order terms may be neglected in the classical regime. For the gauge invariant field strength we have

$$\tilde{\mathcal{F}}_{\mu\nu} = (B + 2\pi\alpha' F)_{\mu\nu} \quad (3)$$

$\alpha'$  denoting the slope parameter in the open bosonic string theory and  $F_{\mu\nu}$  the electromagnetic field tensor. The equation of motion for the gauge field is

$$\partial_\mu \mathcal{F}^{\mu\nu} = 0. \quad (4)$$

The field equations obtained by the variation of the action (1) are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} \quad (5)$$

where the energy-momentum tensor  $T_{\mu\nu}$  is

$$T_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\delta S_{\text{DBI}}}{\delta g^{\mu\nu}} = \frac{1}{2} \left( \frac{1}{4} g_{\mu\nu} \mathcal{F}_{\mu'\nu'} \mathcal{F}^{\mu'\nu'} - \mathcal{F}_{\mu\lambda} \mathcal{F}_\nu^\lambda \right). \quad (6)$$

The uniform electromagnetic field on the brane is expressed by its components  $\mathbf{E} = (0, E_2, E_3)$  and  $\mathbf{B} = (0, B_2, B_3)$ . The U(1) gauge potential is

$$A_\mu = \left( \frac{-i Q_e}{r}, 0, 0, Q_m \cos \theta \right) \quad (7)$$

with  $Q_e, Q_m$  constants denoting the electric and magnetic charge, respectively, while the electromagnetic field takes the (anti-parallel configuration) form

$$\mathbf{E} = -\frac{Q_e}{r^2} \hat{r} \quad \text{and} \quad \mathbf{B} = \frac{Q_m}{r^2} \hat{r}. \quad (8)$$

The effective metric on the brane is given by Seiberg and Witten (1999)

$$G_{\mu\nu} = g_{\mu\nu} - (\mathcal{B} g^{-1} \mathcal{B})_{\mu\nu} \quad (9)$$

with  $\mathcal{B}_{\mu\nu}$  a constant 2-form induced on the world volume of the D3-brane.<sup>3</sup> In the classical regime, for  $\mathcal{B} = 0$ , the action (2) reduces to that of General Relativity coupled to Maxwell's electromagnetism, while (3) becomes  $\tilde{\mathcal{F}}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}$ . Then, the effective metric on the brane reads

$$G_{\mu\nu} = g_{\mu\nu} - (\tilde{\mathcal{F}} g^{-1} \tilde{\mathcal{F}})_{\mu\nu} + \mathcal{O}(\mathcal{F}^4) + \dots \quad (10)$$

<sup>3</sup> In the presence of a D-brane, a constant  $\mathcal{B}$ -field cannot be gauged away and can be reinterpreted as a constant magnetic field on the brane.

Since  $T_{\mu\nu}$  is weak, the gravitational solution can be approximated by the Schwarzschild geometry. Thus, a (semi)classical solution of Eq. (4) and (5) can be obtained and, ignoring higher order terms in (10) as in (2), the line element for the effective metric finally becomes

$$\begin{aligned}
 ds^2 = & - \left(1 - \frac{2M}{r}\right) \left(1 - \frac{Q_e^2}{r^4}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} \left(1 - \frac{Q_e^2}{r^4}\right)^{-1} dr^2 \\
 & + \left(1 - \frac{Q_m^2}{r^4}\right) r^2 d\theta^2 + \left(1 - \frac{Q_m^2}{r^4}\right)^{-1} r^2 \sin^2 \theta d\varphi^2.
 \end{aligned}
 \tag{11}$$

Actually, the metric given in Kar and Majumdar (2005) has a positive sign in front of the first term. However, the negative sign here arises from the transformation  $t \rightarrow it$  in going from the Euclidean to the Lorentzian signature. The above line element describes an, asymptotically flat, Schwarzschild-like geometry, which becomes the known Schwarzschild solution when  $Q_e = Q_m = 0$ . This (semi-)classical solution is of Petrov type I and it is not spherically symmetric. It becomes of Petrov type D and acquires spherical symmetry only when  $Q_m = 0$ . By computing the Kretschmann scalar, one can see that, in the general case, i.e., when the electric as well as the magnetic charge are nonzero, the above solution has two curvature singularities (at  $r = 0$  and at  $r = \sqrt{Q_m}$ ). Furthermore, there are three horizons, namely at  $r = 2M$ , at  $r = \sqrt{Q_e}$ , and at  $r = \sqrt{Q_m}$ . In fact, one of the curvature singularities turns out to be also a horizon. As it is evident, the above solution describes a highly exotic situation, which should be studied further and in more detail.

### 3. MØLLER'S PRESCRIPTION

In the Introduction we have pointed out the importance of the energy-momentum complexes for the energy-momentum localization thereby stressing the role of the Møller prescription in this context. The Møller energy-momentum complex is an efficient tool for the energy-momentum localization and allows obtaining satisfactory results for the energy and momentum distributions in the case of a general-relativistic system.

The Møller energy-momentum complex in a four-dimensional background (Møller, 1958) is given as

$$\mathcal{J}_\nu^\mu = \frac{1}{8\pi} \xi_{\nu, \lambda}^{\mu\lambda}
 \tag{12}$$

where Møller's superpotential  $\xi_\nu^{\mu\lambda}$  is of the form

$$\xi_\nu^{\mu\lambda} = \sqrt{-g} \left( \frac{\partial g_{\nu\sigma}}{\partial x^\kappa} - \frac{\partial g_{\nu\kappa}}{\partial x^\sigma} \right) g^{\mu\kappa} g^{\lambda\sigma}
 \tag{13}$$

with the antisymmetric property

$$\xi_v^{\mu\lambda} = -\xi_v^{\lambda\mu}. \tag{14}$$

It is easily seen that Møller’s energy-momentum complex satisfies the local conservation equation

$$\frac{\partial \mathcal{J}_v^\mu}{\partial x^\mu} = 0 \tag{15}$$

where  $\mathcal{J}_0^0$  is the energy density and  $\mathcal{J}_i^0, i = 1, 2, 3$ , are the momentum density components.

Thus, in Møller’s prescription the energy and momentum for a four-dimensional background are given by

$$P_v = \int \int \int \mathcal{J}_v^0 dx^1 dx^2 dx^3. \tag{16}$$

Specifically, the energy of a physical system in a four-dimensional background is

$$E = \int \int \int \mathcal{J}_0^0 dx^1 dx^2 dx^3. \tag{17}$$

In this prescription the calculations are not anymore restricted to quasi-Cartesian coordinates. They can be utilized in any coordinate system.

#### 4. ENERGY AND MOMENTUM DENSITY DISTRIBUTIONS

First, we have to evaluate the superpotentials. There are eight nonzero Møller superpotentials:

$$\xi_0^{01} = -\xi_0^{10} = -\left[4Q_e^2 \frac{1}{r^3} \left(1 - \frac{2M}{r}\right) + 2M \left(1 - \frac{Q_e^2}{r^4}\right)\right] \sin \theta \tag{18}$$

$$\xi_2^{12} = -\xi_2^{21} = -\frac{2}{r^3} \left(1 - \frac{2M}{r}\right) \left(\frac{Q_m^2 + r^4}{Q_m^2 - r^4}\right) (r^4 - Q_e^2) \sin \theta \tag{19}$$

$$\xi_3^{13} = -\xi_3^{31} = 2r \left(1 - \frac{2M}{r}\right) \left(1 - \frac{Q_e^2}{r^4}\right) \left(\frac{3Q_m^2 - r^4}{Q_m^2 - r^4}\right) \sin \theta \tag{20}$$

$$\xi_3^{23} = -\xi_3^{32} = -\frac{2r^4 \cos \theta}{Q_m^2 - r^4}. \tag{21}$$

By substituting the superpotentials given by (18–21) into (12) we get for the energy density distribution

$$J_0^0 = \frac{Q_e^2(3r - 10M) \sin \theta}{2\pi r^5}. \tag{22}$$

Furthermore, it is found out that all the momentum density distributions vanish. Now, substituting (22) into (17) and evaluating the integral, we obtain the energy contained in a “sphere” of radius  $R$ :

$$E(R) = M - 2Q_e^2 \left( -\frac{5M}{2R^4} + \frac{1}{R^3} \right). \quad (23)$$

This result, depending only on the electric charge, gives the effective gravitational mass for the spacetime considered. At very large distances, i.e. at the asymptotic limit, the energy equals the (ADM) mass  $M$ . However, the energy equals the mass  $M$  also for the finite radius  $R = \frac{5}{2}M$ , an unexpected and remarkable result for which, beyond speculations, no reasonable explanation has been found. At this stage of the investigation, one can only conjecture that this result would be attributed to the highly exotic character of the object having the spacetime described by Eq. (11).

## 5. DISCUSSION

A D3-brane in the presence of a uniform electromagnetic field in an open bosonic string theory is considered. By including a slow variation in the induced metric tensor  $g_{\mu\nu}$  in the classical regime, one ends up with an action describing the D3-brane dynamics and consisting in the coupling of the Einstein-Hilbert action to a Dirac-Born-Infeld action, whereby a static gauge condition is assumed. The aforementioned complete action leads, in the classical regime, to an effective metric describing the geometry of a Schwarzschild-like spacetime on the D3-brane (Kar and Majumdar, 2005) with curious properties that need to be further investigated.

In this work, we have explicitly calculated the energy and momentum densities for this effective metric. The geometry considered is spherically symmetric when the magnetic charge  $Q_m = 0$ . Furthermore, if the electric as well as the magnetic charge vanishes, the geometry is identical with that describing the spacetime exterior to a Schwarzschild black hole. The energy and momentum densities are computed using the Møller energy-momentum complex. It is found that all the momentum densities vanish, while the effective gravitational mass, i.e. the total energy contained in a “sphere” of radius  $R$  in the considered, Schwarzschild-like, spacetime, depends on the mass  $M$  and the electric charge  $Q_e$ . At the asymptotic limit, the energy is equal to the (ADM) mass  $M$ . However, this value is also obtained for a finite radius  $R$ , a result that remains an open question to be answered. Last but not least, the paper sustains Lessner’s argumentation (Lessner, 1996) supporting Møller’s prescription as a powerful tool for describing the concepts of energy and momentum in General Relativity.

Work on the computation of the energy and momentum distributions for the Schwarzschild-like geometry generalized to a black hole solution on a noncommutative D3-brane in a static gauge at the Planck scale, is in progress.

## ACKNOWLEDGMENTS

The authors are indebted to Dr. E.C. Vagenas for stimulating discussions and useful suggestions and to Dr. G.O. Papadopoulos for valuable comments. They, also, thank the unknown referees for their suggestions which helped correcting and improving the final form of the manuscript.

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